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VARIABILITY IN UNIVERSITY MATHMETICS TEACHING: A TALE OF TWO INSTRUCTORS

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This paper examines two lessons in an infinitesimal calculus course given by two instructors with different backgrounds and teaching agendas. The lessons were based on the same lesson plan but decisions the instructors made prior to class took the lessons in substantially different directions. Using Schoenfeld's framework for analysing decision-making processes we describe the resources, beliefs, attitudes and goals that guided and restricted the instructors. These findings provide an insight into the different agendas and considerations underlying the instructors' decisions and the subsequent course taken by their lessons. On the basis of this analysis we will reflect on the impact that background, teaching experience and orientations have on university mathematics teaching. This discussion is a contribution to the increased pedagogical awareness in university mathematics teaching.

INTRODUCTION

Students in university math courses are often divided into groups where the same curriculum is taught concurrently by different instructors. This is the case in a calculus course for first year math students at a leading university in Israel. Aside from the lectures, every week the students divide into small groups and attend a teacher assistant's (TA) lesson of their choice. These lessons are supposed to follow a lesson plan written according to the lecturers' needs and consequently the students are told they can attend any TA lesson because all the TAs teach more or less the same lesson. In this study we examine and compare two of these lessons given by two different TAs and based on the same lesson plan. These two lessons showed substantial differences that were in part a result of decisions each TA made while preparing for his lesson. These decisions are based on different interpretations of the lesson plan that led to the forming of different teaching goals and consequently to different adaptations of the lesson plan. The question this paper reflects upon is: *What led each of the TAs to interpret and implement the lesson plan in the way that he did.*

In recent years various members of the research community noted that there is a shortage of empirical research describing and analysing the practice of teaching mathematics at the university level. Speer et al. (2010) note that only little is known about what university math instructors do and think daily, in class and out, as they perform their teaching work. Weber (2004) states that most of the research that does exist in this topic consists of researchers' suggestions for improving pedagogy and that there are relatively few studies on how advanced mathematical courses are actually taught. In their survey, Speer et al. (2010) list (only) five empirical studies that analyze teaching practices at a sufficiently fine level of detail so that other instructors and researchers can inspect and learn from the instructional choices and reasoning described therein. One of these studies focuses on the beliefs of a novice instructor of college mathematics and their influence on his instructional practices, particularly on his in-the-moment decisions in class (Speer, 2008). In-the-moment decisions of instructors were also the focus of the case studies described in (Hannah, Stewart, & Thomas, 2011) and (Paterson, Thomas, & Taylor, 2011) where Schoenfeld's theory on decision making processes (Schoenfeld, 2011) was used to describe and analyse decision processes that took place during math lectures.

In this study we continue this line of employing Schoenfeld's theory and analysing the resources, orientations and goals (ROG) underlying instructors' decisions. The contribution of this paper is twofold. First, it examines decisions that were made prior to the first TA lessons in the course and that are thus unaffected by many factors which normally influence classroom in-the-moment decisions. Second, we have in front of us decisions made in very similar conditions by two TAs with very different backgrounds: TA1 is a math graduate student with little teaching experience, while TA2 is a very experienced calculus instructor with a PhD in mathematics, who is not engaged in active research. Thus in this paper we do not only describe the effect of the TAs' agendas on their decisions, we also reflect on the impact of the TAs' background and teaching experience on their stated and inferred resources, beliefs, attitudes and teaching goals and consequently on the way their lessons evolved.

Another focus of this paper is the substantial gap between the intended and the implemented curriculums in the TA lessons. While there are many studies about such gaps in the context of primary and secondary math education (e.g. Even & Kvatinsky, 2010), here this phenomenon is examined in the context of university math education.

We shall not discuss here the mathematics that each TA made available to his students, and the impact of the findings of this paper on students' learning. These two important aspects of the TA lessons are the subject of an ongoing follow up study.

THEORETICAL FRAMEWORK

To analyse the decisions made by the TAs we will employ Schoenfeld's theory on in-the-moment decision making processes. Schoenfeld (2011) asserts that what people do is a function of their resources (intellectual, material, and contextual), orientations (their beliefs, values, biases, etc.) and goals (the conscious or unconscious aims they are trying to achieve). According to Schoenfeld these categories are both necessary and sufficient for understanding decision making. Necessary in the sense that if any of these categories is left out of an analysis then the analyst runs the risk of missing a key factor in the decision making process, and sufficient in the sense that every root cause of decision making can be found within these categories. Thus, in order to understand the considerations underlying the TAs' decisions, we should aim to uncover the explicit and implicit orientations and goals that guided each TA as well as the resources they had at their disposal while preparing for class. We retain that the analysis done here does not make use of the full strength of Schoenfeld's theory. The theory addresses decision making processes and requires attending to the factors that shape the teacher's prioritizing and goal setting when potentially consequential unforeseen events arise. These factors are critical when dealing with in-the-moment decisions. However, the decisions we focus on are made prior to class, and are often planned, conscious and deliberate. We will therefore refer to the ROG categories as a framework that guides us where to look while analysing the TAs' decisions.

RESEARCH CONTEXT AND METHODS

This study focuses on the lessons of two TAs: TA1 is an exceptionally bright math graduate student. He has been teaching TA lessons for 4 years in various math courses and has taught calculus only once, 3 years prior to this study. TA2 has been the head TA of this course for more than a decade and he is a very experienced and a very popular calculus instructor. He has a PhD in mathematics but he has not been engaged in mathematical research for several years. A noteworthy fact is that TA1 is not only inexperienced as a calculus teacher but also as a calculus student since he taught himself calculus prior to his university studies and did not attend first year calculus courses as a student. The two lessons examined here were the first TA lessons in this course and revolved around the definition of the derivative that had been taught earlier in the lectures. The lesson plan for these lessons was prepared by a third TA according to the demands of the lecturers and under the supervision of TA2. The TAs in this course were expected to follow this plan but there was no supervision of its actual implementation during or after the TA lessons.

The author attended the lessons as a non-participant observer, audio recorded them and took notes. After each lesson the TAs reflected on their preparation for class. Each lesson was compared to the lesson plan and an initial analysis of the TAs' ROG was conducted by inferring from the TAs' actions in the classroom. Confirmation and expansion of this analysis was made in final interview with each of the TAs.

AN OVERVIEW OF THE TWO LESSONS

The lesson plan revolved around the following definition: *Suppose f is defined in a neighborhood of some point* x_0. *Then we will say that f has a derivative a at* x_0 *if the limit* $\lim_{x\to x_0}(f(x)-f(x_0))/(x-x_0)=a$. The plan had a theoretical part with several observations regarding the definition (e.g. differentiability implies continuity) and a practical part with three exercises, along with concise, formal, algebraic proofs:

A. *Find the derivative of the function* $f(x)=\sqrt{5x+1}$ *at* $x_0=3$.

B. *Show that the function* $f(x)=sign(x)\cdot x^2$ *is differentiable at every point and find its derivative at* $x_0=0$.

C. *Show that if f is differentiable at* x_0 *then* $\lim_{h\to 0}(f(x_0+h^2)-f(x_0))/3h=0$.

The plan did not specify time allocation or any teaching/learning goals.

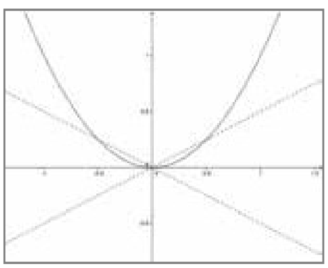
TA1's lesson

TA1 started his lesson by writing the definition of the derivative. Then TA1 stated that he would like to develop some intuition before getting into examples and departed from the lesson plan, initiating a teaching sequence that he designed. TA1 rewrote the definition in terms of epsilon and delta and with a few algebraic steps he obtained the following: *Suppose* $f'(x_0)=a$ *then for every* $\varepsilon>0$ *there exists* δ *such that* $0<|x-x_0|<\delta$ *implies* $(a-\varepsilon)(x-x_0)+f(x_0)<f(x)<(a+\varepsilon)(x-x_0)+f(x_0)$. TA1 then clarified to the students the geometric meaning of this statement:

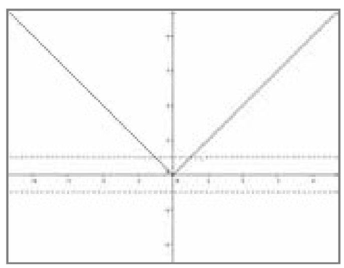
What we see is that if f is differentiable at x_0 *then for any given epsilon the (graph of) f is bounded, in some neighbourhood of* x_0*, from above and from below by two lines with slopes* $a+\varepsilon$ *and* $a-\varepsilon$ *crossing each other at the point* $f(x_0)$*. We are now ready to draw a picture.*

At this point TA1 drew **Drawing1** and used it as a visualisation aid while repeating his previous explanation. He then noted that differentiability implies continuity:

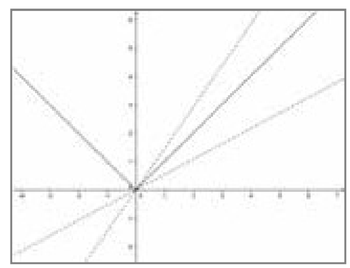
When I get closer to x_0 *(TA1's finger slides on the graph towards* $(x_0, f(x_0))$*) the graph gets closer and closer to the point* $f(x_0)$ *… I'm not giving here a full formal proof; I think that the fact that differentiability implies continuity is very clear from this drawing.*



Drawing1



Drawing2



Drawing3

After this explanation TA1 suggested using the geometric interpretation to develop an intuition as to why the absolute value function is continuous but not differentiable at zero and for this purpose he drew **Drawing2** and **Drawing3**. After discussing with the students what can be seen through these drawings TA1 emphasized that this type of argumentation does not qualify as a formal proof. He set aside the geometric interpretation, wrote on the board a rigorous algebraic proof and concluded:

This is a formal proof. Note however that it actually repeats what we saw in the drawings.

This remark ended the sequence TA1 initiated and he continued to exercise B in the lesson plan. Again he stated that he would like start by developing some intuition. TA1 drew the graph of the function and discussed the notion of "gluing" functions together (in this case x^2 and $-x^2$) and how in some cases (e.g. the absolute value function) gluing differentiable functions yields a non-differentiable function. While addressing the differentiability of the function he introduced the notion of a "local property of a function" and invested a great amount of time discussing why the derivative is indeed a local property. This discussion continued until the end of the lesson and TA1 did not address exercises A or C.

TA2's lesson

TA2's lesson remained close to the lesson plan. Like TA1, TA2 started his lesson by writing the definition of the derivative, while constantly making stops to clarify the mathematical terms he used. Then TA2 advanced quickly through the observations listed in the first part of the lesson plan using them as a springboard for exercise A:

The fact that differentiability implies continuity tells us that if f is differentiable at x_0 then the limit of $(f(x)-f(x_0))/(x-x_0)$ when $x \to x_0$ must be in the form 0/0. Pay attention! This means this limit cannot be found using the arithmetic rules of limits! Let's see an example.

At this point TA2 wrote exercise A on the board, turned to the students and asked:

Is it a legitimate question? Why? Why is f defined in some neighbourhood of 3? How do we start? How shall we find the derivative? Pay attention - Whenever we learn a new concept we always start solving problems by going straight to the definition. Only after some time we start developing theorems that we can use instead of the definition.

After this introduction, TA2 wrote the limit according to the definition and simplified it to obtain: $\lim_{x\to 3}(\sqrt{5x+1}-\sqrt{5\cdot 3+1})/(x-3) = \lim_{x\to 3}(\sqrt{5x+1}-4)/(x-3)$. He then commented:

The claim that the limit is of the form 0/0 relied only on the fact that f is continuous at x_0. Do you agree that f is continuous at 3? So this limit is of the form 0/0. How do we overcome this uncertainty? Recall what we did before in similar situations.

TA1 then showed that $(\sqrt{5x+1}-4)/(x-3) = 5(x-3)/((x-3)(\sqrt{5x+1}+4))$, at which point he turned to the students and raised his voice:

A-ha! (Pointing at the two appearances of $(x-3)$ at the right-hand side) This is the culprit responsible for the fact that both the numerator and denominator tend to zero! We can cancel these expressions $(x-3)$ (which are not zero by definition) and now the limit becomes a simple exercise, since we can use the arithmetic laws of finite limits.

TA2 then continued to solve exercise B, again constantly reflecting on his actions. Unlike TA1, TA2 did not discuss the notion of the derivative as a local property. After exercise B TA2 temporarily left the course of the lesson plan telling the students that he would to review several concepts that were introduced in the lectures. After this review TA2 return to the plan, solved exercise C and concluded his lesson.

PREPARATIONS FOR CLASS

After their lessons the TAs described to the author their preparation for class. TA1 started by reading the lesson plan but after getting to the definition of the derivative he stopped and attempted to *try and make sense of the definition by playing with it and by drawing pictures*. This course of action has led to the geometric interpretation of the definition that TA1 considered interesting and valuable. He then decided to present this interpretation in class. TA1 explained that while reading the lesson plan he was asking in his mind questions like: *What do I find important and interesting in this topic? How do I approach this content? What would help me if I were in the place of my students?*

TA2's preparation was guided by different kind of questions. He explained that while reading the lesson plan he was thinking: *If I were a student, how would I see this topic? What might prove difficult for me? Where will I get confused?*

ANALYSIS

We now turn to describe and analyse some of the decisions the two TAs made.

Decisions

TA1 made on several occasions a conscious decision to deviate from lesson plan, for example when he initiated his original teaching sequence or when he ignored exercises A and C. By contrast, TA2 followed the lesson plan and restricted himself to content that had already been taught in the lectures, in what turned out to be a conscious decision on his part. A tacit decision of TA1 was to precede every rigorous proof with a visual intuition. TA2 did not develop any visual intuition in his lesson and presented to his students strictly rigorous proofs.

The interviews revealed several additional decisions. TA1 teaching was guided by a decision to concentrate on the theoretical aspects of the derivative on the expense of the applicative and procedural aspects. TA2 on the other hand made a decision to advance quickly through the theoretical part of the lesson plan and to concentrate on solving the exercises. TA1 saw in exercise B an opportunity for discussing the notion of the derivative as a local property. TA2 decided to use exercise A to emphasize to the students that the derivation is a property of a function at a point (rather than a property of a function as a whole). Below we expand on and analyse these decisions.

Resources

Both TAs had sound mathematical background and indubitable content knowledge and both relied on the lesson plan as a major resource. However, the TAs' reflections on their decisions uncovered additional resources that they had or lacked.

In his interview TA2 stated that in his opinion *the most significant element in preparation for class is the teaching experience*. Indeed, the rationale TA2 provided for his decisions indicated that his teaching experience played a central role as a resource of pedagogical content knowledge (Shulman, 1986). For example, knowledge of a potential learning obstacle has led TA2's to decide to highlight the notion that the derivative is a property of a function at a point (rather than of a function as a whole):

Students became accustomed over the years to differentiate functions as a whole. You start with $f(x)$ and differentiate it to obtain $f'(x)$. There is usually no mentioning of x_0 anywhere in the process! This approach often causes many difficulties for the students when they encounter functions which are not elementary. Thus it was very important for me to emphasize again and again that student are required to work with the definition, especially in the first exercise where most students would feel temptation to differentiate the function.

TA2's reliance on his experience as a resource was also evident in the rationale he provided for his decision to advance quickly to the second part of the lesson plan:

Keep in mind that in this particular lesson the students have just returned from a long semester break. For this reason I decided to get as quickly as possible to concrete examples, which I know are more effective than abstract theory for getting the students back on track.

In contrast, TA1's lessons were often based on what he himself perceived as interesting or challenging:

While preparing for the lesson I usually encounter something which makes me pause. It can be something I find interesting or maybe some difficulty I have with justifying a certain step in some argument. I often end up taking this something to class, thinking that if I found it interesting or if it got me confused then it would probably do the same to the students. I know it is naïve to think that I and the students would find the same things interesting or confusing, but the way I see it is an inevitable working assumption.

Later in the interview TA1 added that he cannot really say what students in his class need or want. These acknowledgments indicate that TA1's decisions were effected by his lack of pedagogical content knowledge.

Orientations

TA1's reflection on this decision uncovered also some of his beliefs and attitudes regarding how mathematics should be taught and communicated in the TA lessons:

The reason I opened the lesson with the geometric interpretation is that it is not standard. It is a good example of the things I'm drawn to. It is not just textual, it involves drawings and it requires a great amount of explanation. It is not something a student would get just by reading the lecture notes. I believe that the added value of the TA lessons does not lie in well-polished content but rather in the learning experience that it provides for the students.

An important factor in TA1's decisions is the image of the student he had in mind:

I wouldn't say that my teaching is directed at the brightest students but rather the hardest working students. I know there are students who prefer to learn in class just by writing everything that is on the board and then read their notes afterwards in the comfort of their homes or before the exam. I don't claim to provide very good service to these students.

Regarding his decision to focus on exercise B, TA1 expressed a dislike to exercises A and C, stating that he did not find them challenging or interesting:

In the lessons I prefer to focus on the more challenging exercises and let the students struggle with the simple exercises on their own. […] I don't think students benefit much from seeing me just performing algebraic manipulations in front of them. I prefer solutions which represent some mathematic depth, like in exercise B where you have the notion of gluing functions together and the notion of a local property of a function.

Like TA1, TA2's decisions were also shaped by the image he had of his students. In his interview TA2 noted that his decisions often reflect an evaluation of what these students could not manage or could not understand on their own:

I believe I have a responsibility towards all my students, not just those few who will later become mathematicians. The home assignments can be very difficult. I remember myself *struggling with them for hours. I think this is a good thing. However, from my experience this can be a breaking point, especially for the weaker students. I believe that it is the responsibility of the TAs to support these students and to make sure they are not left behind.*

Goals

TA1 reflection on the teaching sequence he designed uncovered an implicit goal:

I wanted the students to leave class with some intuition. The students probably heard in the lectures that the derivative is related to the slope of a tangent line to the graph. This is a nice intuition but there is a big gap between a tangent line, which is something you can draw or visualize, and the formal definition which express the derivative in terms of epsilon and delta. What I tried to do in this lesson is to narrow this gap.

Another goal can be inferred from TA1's actions in class: To model for his students the behaviour of a professional mathematician. This goal was confirmed in the interview, when TA1 reflected on the role and the goals of the TA lessons in general:

I think these lessons should develop the active aspects of learning. How do you approach a task? How do you start a proof? Often before I prove theorems in class I do some preparation. I tell the students that we should first gain some intuition, try and see the big picture of the proof. Same with definitions … As mathematicians, we constantly take actions to gain intuition and one of the goals of these lessons is help students learn how to do that.

An implicit goal, which seems to have had great impact on TA1's decisions, became evident when TA1 reflected on how the TA lessons should complement the lectures:

The lectures are restricted to content which is complete, precise and true. There is an unwritten law that the students can prepare to the exams by reading the lectures notes. I believe that this law does not apply on the TA lessons and that I am obligated to provide the students with a meaningful added value. I want the students to have a different interaction with the content, a mathematical experience that cannot factor through a notebook.

TA2 also addressed the role and goals of TA lessons and their relation to the lectures:

Ideally every new mathematical concept or term introduced in the lectures would be accompanied by several examples. This is not the case and thus one of the main goals of the TA lessons, as I see it, is to review the content taught in the lectures and to give the students an opportunity to get a firm hold on it. […] There are many things that students should hear more than once, preferably from different teachers with different perspectives and different terminologies. You cannot start to imagine how many times I have heard this "ahh" sound of understanding in class after repeating something that was already said in the lectures.

We note other explicit goals of TA2 already mentioned above: Improving students' capability to solve tasks, getting the students back on track after the long semester break, and emphasizing that formally the derivative is a property of a function at a point. The actions of TA2 in this lesson as well as other lessons suggest another goal which guided TA2: To model for the students the behaviour of an idealized student rather than the behaviour of an experienced mathematician. TA2 confirmed this goal.

SUMMARY AND DISCUSSION

There is a well-known myth stating that university math teaching is just a matter of accumulated experience, clear presentation skills and sound content knowledge. One may also speculate that within the community of professional mathematicians working side by side there cannot be much variability with respect to their resources, orientation and teaching goals. In fact, this assumption seems to be institutionalized by telling the students that all the TA lessons are roughly the same. This study not only challenges these assumptions but also highlights and analyses the differences between two instructors, both with sound mathematical background and indubitable teaching proficiency, implementing the same lesson plan. Using Schoenfeld's ROG framework we showed how different beliefs and attitudes, different goals and reliance on different resources resulted in two substantially different lessons.

A more specific contribution of this study refers to the ways in which the TAs' pedagogical content knowledge, or lack of it, affected the lessons. TA2 stated that his teaching experience was his most significant resource in preparation for lessons, highlighting the specific difficulties he expected from his students and ways of addressing them. For example, TA2's experience suggested that students will be better off with concrete examples and led him to decide to advance quickly through the theoretical part of the lesson plan. Similarly, TA2's decision to focus on the notion that the derivative is a property of a function at a point was his way to address what he considered a common mistake of students, and a potential learning obstacle. By contrast, TA1's lack of calculus teaching experience forced him to choose goals according to what he found interesting or difficult, led by his own introspective processes and by his aesthetics and values as a researcher trying to make sense of new concepts and ideas, designing teaching sequences accordingly. TA1 acknowledges that his perspective on the content may not necessarily coincide with the students' viewpoint or needs, but he sees it as his best possible approximation.

Paterson et al. (2011) suggested that lecturers who are research mathematicians bring different, at times conflicting, orientations into play. It is thus interesting to observe and compare the orientations of the two TAs, whose backgrounds represent these two identities. TA1 is first and foremost a researcher, and believes that it is in the best interests of his students that he will display before them the tools of the trade of a mathematician at work. At the same time TA2, who is first and foremost an instructor, believes that it is in the best interests of his students that he will model the behaviour of an idealized student and attend to their potential learning obstacles. TA1 directed his teaching at the mathematically oriented students, even though he acknowledged that by doing that he may not be providing good services to many of his students. TA2, on the other hands, felt a strong commitment to all his students and *not just those few who will later become mathematicians*, but he does so at the cost of not fully attending the needs of the brighter students in his class. While acknowledging these two identities as a potential source for conflicts regarding university math instruction, it is important to note that these two identities may also combine in fruitful ways. In this sense, it would be interesting to study a collaboration between TA1 and TA2, or any other two instructors representing their distinct identities, and examine how and when do their different orientations conflict or synergize.

An interesting by-product of this study was the way it influenced the TAs themselves to reconsider their beliefs and attitudes, simply by having a chance to talk with an outside observer about their teaching. TA1 noted that after reflecting on his lesson and discussing it with the author, he has decided to routinely ask questions and initiate dialogues during class and thus obtain a better understanding of his students' viewpoint on the content. TA2 expressed his worries on whether his teaching has strayed too far in favor of the weaker students at the expense of the rest of the class and especially the brighter students. These outcomes suggest that the approach taken in this study, where instructors reflect on decisions they and other colleagues make under similar conditions, may be used in development programs to enhance teaching.

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