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DIVERSITY IN THE CONSTRUCTION OF A GROUP'S SHARED KNOWLEDGE

Rina Hershkowitz and Nurit Hadas — Weizmann Institute of Science
Tommy Dreyfus — Tel Aviv University

We describe and analyze episodes taken from a long-term research project, whose main goal is to investigate the constructing and consolidating of knowledge in elementary probability. Specifically, we follow the constructing and consolidating of "shared knowledge" by a group of three students in one of the project classrooms. The RBC model is used as the main methodological tool. We found that the group constructed shared common basis of knowledge, which enable them to continue the constructing of a new knowledge. We also found that this knowledge flows from one student to the other, where many times each partner has her own way of constructing knowledge.

INTRODUCTION

The relationships between individual students' knowledge and what might be called the "shared knowledge" of the ensemble is a fascinating issue, both from cognitive and socio-cultural points of views. We consider ensemble in the sense of Granot (1998) as "the smallest group of individuals who directly interact with one another during developmental processes related to a specific context" (p. 42).

However, the researcher who plans to observe and analyze in detail processes of *constructing* knowledge in an ensemble, in a given context, and along a time segment, in which some learning occurs, will face great difficulties: The observation and documentation processes are complicated, data are usually heavy and there is no systematic clear-cut methodology for analyzing them. The individuals' diversity of constructing knowledge within the ensemble is an additional crucial aspect, which makes it hard to define the shared knowledge of the ensemble. All these difficulties grow as the number of the ensemble's participants becomes larger.

Many researchers are aware of the difficulty individual diversity presents for defining shared knowledge. For example, Cobb and his colleagues analyzed the collective learning of a classroom community in terms of the evolution of classroom mathematical practices (Cobb, Stephan, McClain, & Gravemeijer, 2001). For this purpose, they felt the need to coordinate "a social perspective on communal practices with a psychological perspective on individual students' diverse ways of reasoning as they participate in those practices" (p.113). They discussed the notion of *taken as shared activities* of the students in the same classroom, where taken as shared learning is such an activity. The following is their explanation for using the above term:

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> “We speak of normative activities being taken as shared rather than shared, to leave room for the diversity in individual students' ways of participating in these activities. The assertion that a particular activity is taken as shared makes no deterministic claims about the reasoning of the participating students, least of all that their reasoning is identical.” (p. 119).

The main goal of the present paper is to follow and investigate processes of *constructing and consolidating of "shared knowledge"* within an ensemble of students learning together. The data form part of the corpus of data of a long-term research project, whose goal is to investigate the constructing and consolidating of knowledge in elementary probability. Because of the detail needed in order to understand and interpret these processes, we chose to focus on a group of three students in a classroom. Since we are aware to the potential diversity of constructing knowledge processes within the group of three, we will relate to the individuals’ processes of constructing knowledge concerning the learned issue, and to the interactions between individuals, and the flow of knowledge from one student to the other. Than we will emphasize the "group's shared knowledge”, which is the group’s common basis of knowledge within these processes. This common basis allows the three students to continue to work together during further learning activity, in which the consolidation of the shared knowledge might be evidenced. Thus the research focuses on the constructing processes as well as on the constructs at a given point of time, and also on their consolidation, whereby personal diversity and the unique flavor of each individual is observed and analyzed.

We grouped the relevant data in narratives, taken from the activities of various groups from different schools, but all on tasks belonging to the same sequence designed for learning elementary probability. The flow, in which the "shared knowledge" is constructed out of the individuals' knowledge, shows many variations. Some of these are exemplified in one narrative, which we present here.

THE RBC MODEL

The RBC model will be used as the main methodological tool for describing and analyzing the constructing of shared knowledge and its consolidation (Hershkowitz, Schwarz, & Dreyfus (HSD), 2001; Dreyfus, Hershkowitz, & Schwarz; (DHS), 2001). The RBC model is a theoretical and practical model for the cognitive analysis of abstraction in mathematics learning. This model suggests constructing as the central process of mathematical abstraction. Processes of new knowledge construction are expressed in the model through three observable and identifiable epistemic actions, *Recognizing, Building-with, and Constructing* (whence RBC). Constructing of new knowledge is largely based on *vertical re-organizing of existing knowledge constructs* in order to create a new knowledge construct. Recognizing takes place when the learner recognizes that a specific knowledge construct is relevant to the problem s/he is dealing with. Building-with, is an action comprising the combination of recognized knowledge elements, in order to achieve a localized goal, such as the actualization of a strategy or a justification or the solution of a problem. The actions of recognizing and building-with are often nested within the action of constructing.

Moreover, constructing actions are at times nested within more complex constructing action. Therefore the model is called the "nested epistemic actions model of abstraction in context", or simply the "RBC-model". A more detailed discussion of the RBC model may be found in the 2 papers above, in which two case studies of students in laboratory settings were analyzed and led the researchers to initiate the elaboration of the model. After starting with an interview with a single student in the first paper (HSD, 2001), the researchers turned to the observation of dyads working in collaboration in the second (DHS, 2001). In this second case study, the construction of knowledge of the dyad and the construction of a new construct of knowledge of each individual in the dyad were investigated by analyzing interactions between the two students. Interaction was investigated in detail as a main contextual factor determining the process of abstraction. From this point of view, the present article is a continuation of the DHS paper. Since then, The RBC model has been validated and its usefulness for describing processes of abstraction of other contents, and in a variety of contexts has been established by a considerable number of research studies by our group as well as by others (e.g., Bikner-Ahsbahs, 2004; Dreyfus & Kidron, in press; Williams, 2006).

Later studies investigated the consolidation of the new knowledge constructs. Consolidation is expected to occur in learning activities that follow the one in which the new knowledge construct first emerged. Evidence for consolidation might be found in the epistemic actions in these following learning activities. And indeed research showed that the RBC model can be extended to processes of abstraction and its consolidation on a medium term time-scale (Dreyfus & Tsamir, 2004; Tabach, Hershkowitz & Schwarz, in press; Monaghan & Ozmantar, in press).

THE RESEARCH PROJECT IN THE CONTEXT OF PROBABILITY

In our current research project, the focus is on students' learning during sequences of activities with a high potential for constructing and consolidating. The "RBC model" was expanded to the "RBC+C" model, where the second C stands for Consolidation.

It was decided to focus in this project on the basic concepts of probability, for several reasons:

- Probability is part of the 8th grade curriculum; the research thus inserts itself naturally in the activity of the school year and contributes to the learning in the experimental classes.
- The topic of probability has relatively little interaction with other topics.
- Intuition plays meaningful roles in probabilistic thinking, because probability offers many exciting connections to daily life (e.g., Falk, Falk and Levin, 1980; Konold, 1989). Moreover intuition might lead to wrong conclusions and hence to surprises and conflicts (Kahneman and Tversky, 1972). Students' initial knowledge is thus undifferentiated, as described by Davydov (1972/1990) and may become articulated and abstracted in adequate activities.

- The hierarchal structure of probability makes it possible to design a sequence of tasks that offers opportunities for constructing of a set of concepts and processes and their consolidation.

A unit consisting of a carefully designed sequence of activities for learning elementary probability has been developed and used with pairs of students as well as in classrooms. The unit includes:

- (i) A written pretest to be answered individually;
- (ii) Five activities (about ten lessons), constructed as sequences of problem situations for group investigations, for whole class discussions, and for (mostly individual) homework assignments;
- (iii) Three post-tests: a written post-test, an interview, and a game/interview, all to be carried out individually.

THE STUDY

Five different teachers taught the unit in eighth grade classrooms, in four different schools. The regular classroom work included group work, whole class discussions led by the teacher, teacher demonstrations, homework discussion, and tests. In each class, a focus group of two or three students was chosen by the researchers and the teacher. The choice criteria were average ability and good verbalization.

In each lesson one or two researchers were present, and documented the lesson by means of two video cameras. One camera focused on the focus group along all the unit's lessons, and the second camera focused on the teacher and the activity of the class as a whole. The researchers also took field notes and collected students' written work.

Having access to a group's work over all ten lessons of the unit allowed us to focus on consolidating processes in addition to constructing of knowledge processes. For the purpose of analysis, narratives concerning the construction and consolidation of knowledge were chosen for groups working in different classrooms but on the same task sequence. Particular attention was paid to the social interactions (group, student-teacher) and frameworks (whole class, small groups, individuals) within which the epistemic actions occurred. Because of space limitations, we will here present and analyze only one narrative from one group.

CONSTRUCTING 2D SAMPLE SPACE – PRINCIPLES AND TASKS

The unit deals with the overall construct of Sample Space, and is organized in three hierarchical stages:

- I. Sample Space in one dimension (1d SS). A simple event in such a sample space is, for example, to obtain 3 when throwing a die.
- II. Sample Space in two dimensions (2d SS), for cases where the possible simple events in each dimension are equi-probable; in such cases, the 2d simple events (expressed as pairs) can be counted and organized in a table,

and the probabilities of complex events can be counted or calculated from the table. A simple event in such a sample space is, for example, to obtain 3 and tails, when throwing a die and a coin.

- III. Sample Space in two dimensions, for cases where there might be a few possible simple events in each dimension, which are not necessarily equi-probable, but whose probabilities are explicitly given; in such cases, the 2d simple events can be organized, in an area diagram, from which the probabilities of more complex events can be calculated.

The data in this paper will be taken from students' activity in stage II of the learning unit and mainly concern one epistemic principle of 2d SS, which we call principle E1: *A simple event in 2d SS consists of a pair of simple events, one in each dimension*. Example: The possible outcomes on each of two dice create pairs of numbers as simple events. In 2d SS, constructing E1 is a necessary condition for constructing other principles, for example principles concerned with the collection of all possible simple events (E2) and with the relevant events for a particular problem situation (E3).

The following probability tasks from Activity 3 (Q1 & Q2) were used in this study:

Activity 3, Q1:

1a Yossi and Ruthie throw two white dice. They decide that Ruthie wins if the numbers of points on the two dice are equal, and Yossi wins if the numbers are different.
Do you think that the game is fair? Explain!

1b The rule of the game is changed. Yossi wins if the dice show consecutive numbers.
Do you think the game is fair now?

Activity 3, Q2:

2 We again throw 2 regular dice. This time we observe the difference between the bigger number of dots and the smaller number of dots on the two dice. (If the numbers on the two dice are equal, the difference is 0.)

Make a hypothesis whether all differences have equal probability. Explain!

It is important to note that activity 3 is the first one in the unit, which deals with 2d sample space, and hence Q1 and Q2 were the first time the students in the study dealt with 2d sample space. (Activities 1 and 2 deal with 1d sample space.) Even more importantly, our epistemic analysis showed that there is no way to deal with these questions without constructing E1.

CONSTRUCTING E1 TOGETHER

In this narrative, the discourse among three girls, Yael, Rachel and Noam, shows how shared knowledge concerning E1 is constructed. The three girls start by discussing Q1a. Yael counts pairs of numbers and reaches 27 pairs for Yossi and Ruthie together.

21 Noam What are you doing?

22	Yael	I did: What are the chances… (1,1) (2,2) (3,3) (4,4) (5,5) (6,6) and (1,1) (1,2) (1,3) (1,4) and (2,1) (2,2).

It seems that Yael is already busy in counting all the relevant pairs for Ruthie's and Yossi's chances to win (E3), and wants to know what are all possible events (E2). We don't know when and how Yael has constructed the meaning of a simple event in 2d Sample Space (E1). In any case, she seems to recognize E1 right from the beginning of her work on Q1, while trying to build-with it further constructs.

Rachel and Noam haven't yet constructed the meaning of E1, of simple events as pairs of numbers; thus they don't understand what, how, and why Yael is counting. Yael does not realize that her friends are not aware that events in 2d SS are represented by pairs (23-27):

23	Rachel	I don't understand what you are doing?
24	Yael	It is because I have to know what is our whole, like: What are all the possible outcomes that might be, and all these outcomes are either (1,1) (2,2) (3,3) (4,4) (5,5) (6,6) or (1,2) (1,3) (1,4) (1,5) (1,6) and then 2 [meaning that now she has to count the combinations of 2 with the other numbers].
25	Noam	Yael, I don't understand anything of what you are doing.
26	Rachel	Nor am I.
27	Yael	Listen, there are some possibilities that 1 will appear: (1,2) (1,3) (1,4) (1,5) (1,6) and we finished with 1, now 2: (2,3) (2,4) (2,5) (2,6).
28	Rachel	O.K., O.K. we understood that, but why are you adding? I don't understand.

Although it is not evident from Rachel's utterances, that she has already constructed E1, Rachel may already share with Yael that one has to count pairs, because she only asks about Yael's conclusion concerning the number of events in the sample space, and not about the nature of simple events in 2d SS. More convincing evidence for Rachel's construction of E1 follows (Rachel 64).

Along the above discussion, Noam expresses objection without any evidence for understanding. She is still in the process of constructing E1, and manages to complete it by confronting her friends with her misunderstanding:

58	Noam	Look you don't… you did as if one side of the die is 3 and the second side is 4 and you did 3 plus 4 and it is as if…
59	Yael	I didn't do 3 plus 4. I will tell you exactly what I did…
60	Noam	No, one second, second. That's what I understood of what you did.
61	Yael	I will explain…

But Noam now wants to explain herself:

62	Noam	One minute! No! You have to do 3 and 4 it is one possibility, and 4 and 5 is a second possibility, so it is two [possibilities].
64	Rachel	That is what she did; (3,4) is one possibility and (5,4) is one possibility.

Noam now appears to have constructed E1 (62), but she still does not represent it as pairs. Rachel (64) provides additional evidence for her own constructing of E1. (It is the first time for Rachel to speak in terms of pairs.).

Additional evidence for the fact that Noam has constructed E1 is that later, while the three girls' work on Q2, Noam explains what is the meaning of the "differences of outcomes" on the two dice:

109	Noam	… as if we look at the difference of one die and the other die; (2,2) then the difference is 0.
110	Rachel	And if we have 1 and 5, then the difference is 4.

We may see here, that after a while, in a later activity (Q2), Noam as well as Rachel recognize the pair nature and its formal representation for simple events of the 2d SS (E1) and use it for building-with it the explanation for the meaning of the differences (Noam 109 and Rachel 110). Thus both of them gave evidence for consolidating E1.

CONCLUDING REMARKS

Constructing and consolidating: Each girl's individual knowledge of E1 was constructed and seems to be consolidated. Yael showed from the very beginning of the group common work on Q1a, that she has E1 construct. But, the questions of Noam and Rachel, the explanations of Yael, and the self-explanations of Noam and Rachel, in the course of constructing this knowledge, have a crucial role. Examples:

1. The insistent questions of Noam and Rachel, led Yael to repeat her counting pairs. While organizing the counting, Yael is producing evidence for her consolidating of the E1 principle (21-27), while counting all possible events and relevant simple events (E2 & E3).
2. Noam puts the blame of her mistakes on Yael (58), and then accepts Yael's refutation (60). This leads her to explain in her own words that she has to relate to pairs (62).
3. Noam and Rachel (109, 110) provided evidence for their consolidation of E1 when using E1 for explaining the meaning of the differences.

In the analyses of the above narrative we exemplified some of the effects the three girls had on each other's constructing and consolidating of knowledge.

"Shared knowledge": As we exemplified at the data above, the three girls now constructed and consolidated E1 and used it in a new task (Q2). Thus E1 is the "shared knowledge" which enable them to continue working together.

We also showed how this knowledge flowed from one girl to the other, where many times each partner has her own way of constructing knowledge, which evolves from a different need, at a different point of time, and also the construct of each individual might varied from one individual to the second, (for example, Noam's informal representation of pairs).

In short, at this point in time principle E1 appears to be a shared common basis of knowledge for the group, and the group may continue the constructing of a new knowledge and/or may show the consolidation of this knowledge in follow up situations.

References

Bikner-Ahsbahs, A. (2004). Towards the emergence of constructing mathematical meanings. In M. J. Høines & A. B. Fuglestad (Eds.), *Procs. PME 28*, Vol. 2 (pp. 119-126). Bergen, Norway: Bergen University College.

Cobb, P., Stephan, M., McClain, K., and Gravemeijer, K., (2001). Participating in classroom mathematical practices. *The Journal of the Learning Sciences 10*, 113-163.

Davydov, V. V. (1972/1990). *Types of generalization in instruction: Logical and psychological problems in the structuring of school curricula* (Volume 2 of J. Kilpatrick, Ed., J. Teller, Trans.: Soviet Studies in Mathematics Education). Reston, VA: NCTM.

Dreyfus, T., Hershkowitz R., and Schwarz B. B. (2001). Abstraction in context II: The case of peer interaction. *Cognitive Science Quarterly 1*, 307-368.

Dreyfus T., and Kidron I. (in press). Interacting parallel constructions: A solitary learner and the bifurcation diagram. *Recherches en Didactique des Mathématiques*.

Dreyfus, T., and Tsamir, P. (2004). Ben's consolidation of knowledge structures about infinite sets. *Journal of Mathematical Behavior 23*, 271-300.

Granot, N., (1998) Unit of analysis in transit: from the individual's knowledge to the ensemble process. *Mind, Culture and Activity 5*, 42-66.

Falk, R., Falk, R., and Levin, I. (1980). A potential for learning probability in young children. *Educational Studies in Mathematics 11*, 181-204.

Hershkowitz, R., Schwarz, B. B., and Dreyfus, T. (2001). Abstraction in context: Epistemic actions. *Journal for Research in Mathematics Education 32*, 195-222.

Kahneman, D., and Tversky, A. (1972). Subjective probability: A judgment of representativeness. *Cognitive Psychology* 3, 430-454,

Konold, C. (1989). Informal conceptions of probability. *Cognition and Instruction 6*, 59-98.

Monaghan, J., and Ozmantar, M.F. (in press). Abstraction and consolidation. *Educational Studies in Mathematics*.

Tabach, M., Hershkowitz, R., and Schwarz, B. B. (in press). Constructing and consolidating of algebraic knowledge within dyadic processes: A case study. *Educational Studies in Mathematics*.

Williams, G. (2006). *Improving the intellectual and affective quality in mathematics lessons: How autonomy and spontaneity enable creative and insightful thinking.* Unpublished PhD thesis, University of Melbourne.