Knowledge Shifts and Knowledge Agents

KNOWLEDGE SHIFTS AND KNOWLEDGE AGENTS

Rina Hershkowitz[*], Michal Tabach[+], Chris Rasmussen[~], and Tommy Dreyfus[+]

[*]The Weizmann Institute of Science, [+]Tel Aviv University, [~]San Diego State University

To better understand the mechanism of knowledge shifts in a learning classroom, we combined two approaches/methodologies that are usually carried out separately: The Abstraction in Context approach with the RBC+C model and the Documenting Collective Activity approach with its methodology. This combination revealed that some students functioned as Knowledge Agents, meaning they were active in shifts of knowledge among individuals in a group, or from one small group to another one, or from their group to the whole class or within the whole class. The teacher as an orchestrator of the learning process is responsible to provide a learning environment that affords argumentation and interaction, in order to enable normative ways of reasoning to be established and to enable students to be active and become knowledge agents.

INTRODUCTION

Tracing students' knowledge construction and the shifts of the constructed knowledge in a working classroom are challenges that still need to be achieved (Saxe et al., 2009). This paper is an attempt in that direction, analysing part of a lesson as a paradigmatic example. We use the theoretical framework of Abstraction in Context (AiC) and the RBC+C methodology to analyse construction of knowledge of individuals, mainly while they are cooperating in small groups in a mathematics classroom; and we use the Documenting Collective Activity (DCA) approach to analyse whole class discussions with the aim of identifying practices that become normative in the classroom. The present study combines the theoretical and methodological aspects of these two lines of research that are usually carried out separately. The goal of the study presented in this paper is to learn more about how knowledge shifts between the different social settings in a working mathematics classroom and the role of individuals and the teacher in these learning processes.

THEORETICAL AND RESEARCH FRAMEWORK

Abstraction in Context (AiC) is a theoretical framework for investigating processes of constructing and consolidating mathematical knowledge (Hershkowitz, Schwarz, & Dreyfus, 2001). Abstraction is defined as an activity of vertically reorganizing (Treffers & Goffree,1985) previous mathematical constructs within mathematics and by mathematical means, interweaving them into a single process of mathematical thinking so as to lead to a construct that is new to the learner.

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The genesis of an abstraction passes through a three-stage process, which includes (i) the need for a new construct, (ii) the emergence of the new construct, and (iii) the consolidation of that construct. A central component of AiC is a theoretical - methodological model, according to which the emergence of a new construct is described and analyzed by means of three observable epistemic actions: *Recognizing* (R), *Building-with* (B) and *Constructing* (C). Recognizing refers to the learner seeing the relevance of a specific previous knowledge construct to the problem at hand. Building-with comprises the combination of recognized constructs, in order to achieve a localized goal. The model suggests constructing as the central epistemic action of mathematical abstraction. Constructing consists of assembling and integrating previous constructs by vertical mathematization to produce a new construct (see Schwarz, Dreyfus and Hershkowitz, 2009 for a detailed description). Students' consolidation of the new construct is revealed when they use this construct for constructing a later one. Consolidation of a construct by a student is characterized, among others, by their progressively quicker recognition of when and where the construct is relevant (immediacy) and by their progressively more flexible building with the construct in changing contexts (Dreyfus &Tsamir, 2004). AiC researchers usually carry out an a priori analysis of the tasks proposed to the students in terms of the knowledge elements that are necessary or useful for successfully completing the task.

Collective activity is a sociological construct that addresses the constitution of ideas through patterns of interaction and is defined as the normative ways of reasoning that are developed in a classroom community. Such normative ways of reasoning emerge as learners solve problems, explain their thinking, represent their ideas, etc. A mathematical idea or a way of reasoning becomes normative when there is empirical evidence that it functions in the classroom *as if it were shared*. The phrase "function as if shared" is similar to "taken as shared" (Cobb &Bauersfeld, 1995) but is intended to make a stronger connection to the empirical approach which uses Toulmin's (1958) model of argumentation to determine when ideas function in the classroom as if they are mathematical truths (Rasmussen & Stephan, 2008).

The DCA methodology begins by using Toulmin's model to create a sequence of argumentation schemes of every whole class discussion, resulting in an argumentation log. In brief, the core of Toulmin's argumentation model consists of three parts: the data, the claim, and the warrant. If one disagrees with the claim, he or she may present a rebuttal, or counter-argument that shows disagreement. When this type of challenge is made, often a qualifier is provided, which is a way to provide specific conditions in which the claim is true. Finally, the argumentation may also include a backing, which demonstrates why the warrant has authority to support the data-claim pair.

The next step involves taking the argumentation log as data itself and looking across all class sessions to see what mathematical ideas become part of the class' normative ways of reasoning. Rasmussen and Stephan (2008) and Cole et al. (2012) identified three criteria for determining when ideas function as if shared. These three criteria can be thought of as the collective analogue to an individual's process of vertical mathematization. In a DCA analysis it is most often the case that one needs to analyze multiple class sessions to find evidence of the three criteria (e.g., Stephan & Rasmussen, 2002). This is because the three criteria capture functional and structural changes to elements of an argumentation over time. In this report we analyze only one episode and hence it would be premature to expect much if any evidence for normative ways of reasoning. As a prelude to such a more comprehensive analysis, we carefully examine, with the aid of Toulmin's scheme, the whole class discussion.

The concepts of knowledge agent & uploading and downloading of ideas

A *knowledge agent* is a member in the classroom community who initiates an idea, which subsequently is appropriated by another member of the classroom community. Thus, when a student in the classroom is the first one to express an idea according to the researchers' observations, and others later express this idea, then this student is considered to be a knowledge agent. Shifts of ideas may be observed from a group to the whole class (*uploading*), or within the whole class, or within a group, or from a group to another group, or from the whole class to a group (*downloading*). Shift actions may occur in different time intervals, which can last from seconds to a few lessons. In the present study we aimed at elaborating the role and function of knowledge agents and the role of the teacher in creating an environment that creates opportunities for students to function as knowledge agents.

METHODS

This study was carried out in the framework of a larger project that involved six grade 8 classes who were engaged in learning probability. The data for this study were collected by video recording in one class. The camera was focused either on the whole class discussion, or on a focus group. A unit consisting of a sequence of problem situations embedded in a rich learning environment was designed and implemented. The activities were carefully designed to offer opportunities for constructing and consolidating knowledge and practices in classroom. The unit included about ten lessons.

The present paper focuses on lesson 4 of the unit. The topics considered in lessons 1 to 4 concerned probability of events in one dimensional sample space and included theoretical probability as a ratio of the number of relevant outcomes to the number of all possible outcomes, as well as some experience with the fact that empirical probability values tend to the theoretical value as the number of trails becomes larger. Lesson 4 starts with a Whole Class (WC) discussion followed by small group work, during which we followed the work of a focus group (FG). In the WC, the teacher initiated a discussion on the *chance bar* and its meaning. The WC discussion was focused on *the Dreidel Problem* and the FG discussion was focused on *the Coin Problem* (see below). Both problems dealt with a qualitative appreciation the probability of composite events. Also, in both problems the chance bar had a qualitative role only.

The Dreidel Problem

A Dreidel is a special kind of top, used as a traditional children's toy. After it is spinning it can fall on one of four sides with equal probability. A Hanukkah dreidel (with four letters N, G, H, and P on its sides) was spun 100 times. Mark approximately on the chance bar the chances of the following events: (a) The dreidel will fall 100 times on the letter N; (b) The dreidel will never fall on the letter N; (c) The dreidel will fall on N between 80 and 90 times; (d) The dreidel will fall on N between 23 and 30 times.

The Coin Problem

A coin was flipped 1000 times. Mark on the chance bar the chances of the following events: (a) The coin will land 1000 times with heads facing up; (b) The coin will land with heads facing up more than 450 times and less than 550 times; (c) The coin will land with heads facing up more than 850 times and less than 950 times; (d) The coin will fall 1000 times with tails facing up.

A priori analysis of the problems

In this paper we will focus only on part d of the dreidel problem and on part b of the coin problem. The following Knowledge Element (KE) was intended to be developed: The chance of an event to fall into a given range of values, which includes the expected value, is high.

FINDINGS

In the following we bring analysis of excerpts from the whole class discussion followed by an analysis of excerpts from the focus group discussion.

Whole Class (WC) episode on the Dreidel Problem (d)

In the episode's protocol we code the participants' utterances using Toulmin's Model (1958): Data – [D]; Claim – [C]; Warrant – [W]; Qualifier – [Q] & Rebuttal – [R]. Due to space constraints only a portion of the episode is presented. In the full episode there were 46 turns, 22 of which were the teacher’s utterances.

128 Teacher	Let's look at d. d says, a top is spun 100 times it will fall on N between 20 to 30 times [D]. What do you think, we will spin the top 100 times, how many times will it fall on N between 20 to 30 times. [To Eliana] Come, you haven't marked yet. (Eliana approaches the board and marks on the chance bar close to the middle [C]).
129 Teacher	Adin, what is your opinion, what do you say?
130 Adin	I think that it is approximately 30% [C].
131Teacher	that means that you agree with what Eliana is suggesting, explain why!
132 Adin	It has more of a chance…
133 Teacher	So if it has more of a chance you are marking it on the 30, more chance for what?

134 Adin — More of a chance than a, b and c [D]. There is a higher chance that it will happen; it is closer to the middle [C].

135 Teacher — So if there is a higher chance you are marking it close to what? Does anyone feel different, want to support or oppose? ... What do you think Guy?

136 Guy — I think it is much higher (Teacher asks how high?) 80% [C], because in fact there are 4 sides to the top [D], right? And the chances that it will fall on one of them is 25% [W] and you said that it will fall between 20 to 30 [D], so…

137 Yael — That means that it is 25% not 80% [R]

…

141 Omri — What I am trying to see is if I understood Guy: what he is trying to say is that there is a 1 out of 4 chance [D], that means that it is a very high percentage that it will be between 20 to 30 [C]

...

144 Teacher — Can you explain again why you are supporting Guy?

145 Omri — What he's saying (Guy) is that every time you spin there is a 1 out of 4 chance that it will fall on N [D], meaning, 25% now out of 100 is approximately the number of times it will fall on the N [C], because it is 1/4 out of four [D]

146 Teacher — What do you think? You are nodding yes (turns to Rachel), who do you agree with?

147 Rachel — With Guy [C]

…

158 Matan — I think it is 65% [C], because it can be more or less, there is a chance that it will come out and a chance that it won't [D]

159 Teacher — But you think it is more than half but a bit lower

160 Matan — Yes

161 Yael — I am still not sure, Guy succeeds to convince me. But at first I thought it was half [C], but still…

162 Teacher — You are still not convinced

163 Yael — I am not sure

164 Teacher — Let me ask you this, let's say that you spin the top 100 times and count how many times it will fall on N, what result would you expect?

...

169 Yael — 25! [C]

170 Teacher — That means that you are expecting an answer between 20 and 30, that is what we are expecting will happen [W]. So if it is what we are expecting that will happen so the chance is close to 1 [C].

DCA analysis: As expected, we did not find evidence at this point for any normative ways of reasoning but conjecture that Guy's reasoning in line 136 is a strong contender for a way of thinking that is likely to become normative. Moreover, our coding of the full WC episode reveals nine different arguments that were generated by the students. Only at the end of the episode (line 170) did the teacher add, mathematically speaking, to these arguments. The high number of student arguments revealed by the Toulmin coding prompted us to look more closely at the role of the teacher in fostering an environment that led to so many student generated arguments. In brief, we identified the following three distinct types of contributions from the teacher: Eliciting student reasoning and justification (e.g., line 129); seeking comparisons of student's reasoning (e.g., line 146); clarifying or re-voicing student reasoning (e.g., line 159). In total, we identified nine turns or portions of a turn in which the teacher elicited student's reasoning, five turns or portions thereof in which the teacher sought comparison of student reasoning, and eight turns or portions thereof in which the teacher clarified or re-voiced student reasoning.

KA analysis: In 136 Guy expresses for the first time in this episode a full argument (Data, Claim and Warrant) concerning the intended knowledge element. Omri in 141 and 145 follows him in different words. Additional students are joining (Rachel 147; Yael 161 with some doubts). In fact the teacher herself in 170 closes the discussion related to Guy’s argument. Thus we can identify Guy as a knowledge agent in this episode. We argue that Guy’s role as a knowledge agent was contingent upon the three different teacher's contributions that elicited, compared, and clarified student reasoning.

Focus Group Episode on the Coin Problem (b)

The focus group includes three girls: Yael, Rachel & Noam.

227	Yael	(reads event b) "will fall on tails more than 450 times and less than 550". It's logical.
228	Noam	It's at half!
229	Yael	No, there are much higher chances (marks b close to 1on the chance bar.)
230	Rachel	That's right, she's correct (supports Yael)
231	Yael	That's what Guy just explained
232	Noam	Right

The discussion on event b took only 6 turns and resulted in agreement between all three students. Yael reads the question and immediately recognized the claim in the question as “plausible” (227). While Noam claims that the chance is half, Yael objects and marks the chances on the chance bar near one. Rachel backs Yael, with no explanation, and Yael provides support – “that’s is what Guy just explained” (231). Noam responds “right” (232). We claim that here we have evidence that Yael (and possibly the other FG's students as well) consolidated the intended knowledge element, after constructing it during the preceding WC discussion: The evidence for constructing it during the WC discussion is strong for Guy and others but only weak for the FG students (Rachel in turn 147 and Yael in turns 137 and 161). However, the present FG discussion shows that at least Yael did construct it then, and that she consolidated it during the present FG discussion. We interpret Yael's turns 229 and 231 as showing consolidation because she uses the construct with immediacy and flexibility, adapting it to a new context, which is characteristic of consolidation. Here we have also clear evidence that Guy was a knowledge agent for Yael, whose ideas were downloaded into the FG discussion. The fact that this group used a way of reasoning from a previous argument as data for a new claim also has strong connections to one of the three criteria for determining when ideas function as if shared (see Rasmussen & Stephan, 2008, for elaboration of the three criteria).

DISCUSSION

In this study, we offer a way to adapt existing methodological tools in order to coordinate analyses of the individual, the group and the collective in a working mathematical classroom. The proposed combined analytic approach is significant in that it offers a new means by which to document the evolution of mathematical ideas in the classroom, the processes by which ideas move between individuals, small groups, and the whole class, and the role of the teacher in these processes. This way helped us to identify shifts of knowledge in the classroom and the students who are main players in these shifts – the knowledge agents. Identifying the role of the teacher in the knowledge construction process and the function of students who act as knowledge agents helps us to understand the mechanism by which collaborative learning can take place, and be affected by individuals and groups in the collective.

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