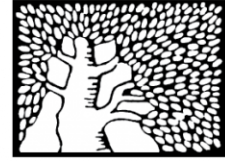


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ALTERNATIVE THEORETICAL PERSPECTIVES IN AND FOR MATHEMATICS EDUCATION

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It is commonly accepted nowadays that a thorough understanding and improvement of the complicated practice of teaching and learning mathematics requires the use of more than one theoretical perspective. Yet, seldom does a single study include analysis of a set of data using different theoretical perspectives. In the following I exemplify what might be entailed and what might be gained in doing so, demonstrating challenges associated with formulating research questions, designing research methodologies, and interpreting results, as well as the potential of such studies for advancing the field of mathematics education.

Theoretical perspectives, multiple theoretical perspectives, mathematics education research, activity theory, verbal analysis.

THE NEED TO USE MORE THAN ONE THEORETICAL PERSPECTIVE

Cognitive studies of students and learning have been part of research in mathematics education for almost four decades. In the 1990's, the focus of research in mathematics education has extended from the individual student's cognition and knowledge to include also contextual, socio-cultural and situated aspects of mathematics learning and knowing. The practices and culture of the classroom community (e.g., the nature of social engagements and norms) have become an important factor in studying learning processes, and mathematics education researchers started to incorporate the two perspectives – cognitive and socio-cultural – into a complex view of mathematics learning (e.g., Cobb, Stephan, McClain and Gravemeijer, 2001). This newer focus signaled a shift from examining human mental functioning in isolation to considering cultural, social, institutional and historical factors as well. The mathematics education community increasingly embraced the view that, like cognitive aspects, cultural and social processes are integral components of mathematics learning and knowing.

Today, it is commonly accepted by researchers in mathematics education that a thorough understanding and improvement of the complicated practice of teaching and learning mathematics requires the use of more than one theoretical perspective. For example, Cobb, Yackel and their colleagues, who have been engaged in a prominent long-term research and development project that aimed at facilitating students' mathematical learning in the classroom, reported that they began the project intending to focus on learning primarily from a cognitive perspective. However, it became apparent to them that they needed to broaden their interpretative stance by developing a sociological perspective as well (Yackel

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& Cobb, 1996). Likewise, in an instrumental article Sfard (1998) proposed two metaphors for learning – the acquisition metaphor and the participation metaphor – warning against the dangers of choosing and adhering to only one: “too great a devotion to one particular metaphor can lead to theoretical distortions and to undesirable practices” (Sfard, 1998, p. 4). Similarly, when addressing the issue of teacher knowing about student learning, Even and Tirosh (2008) argued that,

[C]hoosing and being completely loyal to one learning perspective is counter-productive in educational practice. Adherence to one theoretical perspective might seem an advantage as it eliminates confusion and contradictions. But the task of teaching is much too complex to be reduced to clear-cut global principles, to be applied in all circumstances (p. 214).

Nowadays, a variety of theoretical perspectives, cognitively oriented theories as well as socio-cultural, are commonly used in research studies in mathematics educations. The foci of such research studies range from the individual student’s cognition and knowledge to student and teacher participation in learning and teaching activities, and to different kinds of interaction – between teaching and learning and between knowledge and practice. Yet, rarely does a single study include analysis of a set of data using different theoretical perspectives. In the following I use such a research study – conducted in collaboration with Baruch Schwarz (Even & Schwarz, 2003) – to exemplify what might be entailed and what might be gained in doing so. The next section demonstrates challenges associated with formulating research questions, designing research methodologies, and interpreting results, as well as the potential of such studies for advancing the field of mathematics education.

USING MORE THAN ONE THEORETICAL PERSPECTIVE TO ANALYZE THE SAME SET OF DATA

When exploring the issue of interdependency of theory and research findings, Even and Schwarz (2003) used two theoretical perspectives to analyze a mathematics lesson. The lesson was part of an introductory course on functions for 9th grade Israeli students, in which the curriculum developers aimed for students to investigate problem situations with computerized tools, raise hypotheses, collaborate on solving problems, explain and discuss their solutions, and reflect on their learning in individual and collective written reports (Hershkowitz et al., 2002). The lesson selected for analysis was designed for students to learn about different representations of functions, and of ways to use the graphic calculator to solve problems that require the passage from one representation to another, aiming to create a need to move between symbolic and graphic representations of functions, and develop the ability to do so.

The wide-ranging research aim of the study was:

- *To evaluate whether the intention to create the need, and to develop the ability, to move between symbolic and graphic representations was attained.*

The two theoretical orientations chosen for the analysis of the lesson are common in research in mathematics education, namely, cognitive and socio-cultural. For the cognitive

approach Verbal Analysis (Chi, 1997) was chosen, and for the socio-cultural approach – Activity Theory (Leont’ev, 1981).

Each theoretical perspective indicates a different research paradigm, “the basic belief system or worldview that guides the investigator, not only in choices of method but in ontologically and epistemologically fundamental ways” (Guba & Lincoln, 1994, p. 105). Hence, each theoretical perspective is associated with particular assumptions and expectations (explicit and implicit) related to the nature of learning, knowledge, knowing and understanding, that shape what is being investigated and why, adequate research questions, appropriate units of analysis, adequate length of a study, and so forth. For example, when attempting to formulate the specific research question to be addressed, it became apparent that no research question could conform to both theoretical orientations. Adopting a cognitive approach implied a focus on mental representations, abilities, or capacities, whereas a socio-cultural approach entailed a focus on interactions between developing individuals and the culture in which they live. Adapting to the specific approaches selected, two research questions were formulated, one for each theoretical perspective:

Research question – Cognitive perspective (Verbal Analysis):

- *To what extent do students conceive the passage to a new graphical representation of a function as a problem solving strategy during the lesson?*

Research question – Socio-cultural perspective (Activity Theory):

- *What is the nature of passing from one representation to another in the activity in which the teacher and the students participated during the lesson?*

Similarly to the unsuccessful attempt to formulate one research question to suit the two selected theoretical perspectives, there was also a need to design, for each research question, a research methodology that is aligned with the corresponding theoretical perspective – research methodologies that use the same set of data.

Verbal Analysis of the lesson followed the steps suggested by Chi (1997). The videotaped lesson was transcribed and the transcript was treated as a protocol. To do that, the analysis of the lesson was restricted to the whole-class teacher-centered discussion parts of the lesson only. This enabled the considering of the whole group of students as one entity interacting with the teacher. Segmenting the reduced protocol was based on the passage from one function representation to another, and on the way the representation was used. Each segment of the protocol was coded according to: (1) the representation that was its focus, (2) whether the representation was embedded in a context, (3) who initiated the move to a new representation (i.e., the teacher or the students), (4) who triggered this move, (5) what was the nature of the response to this move (e.g., elaboration, opposition, etc.) and who made it, and (6) the types of utterances contained in the segment (e.g., presentation, short questions, extended answers, etc.) and who formulated them. The coded data were then depicted in two ways, and pattern and coherence were sought in the depicted data, interpreting the data in line with a cognitive perspective.

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As in the case of the Verbal Analysis, data sources for the Activity Theory based analysis included the videotaped lesson and its transcript, and also its verbal analysis and the two ways of depicting the coded data. Yet, whereas the last sources were products of the Verbal Analysis, they were used as artefacts for the Activity Theory based analysis, without necessarily adopting the interpretations of the cognitive perspective. Moreover, unlike the choice made in the case of the Verbal Analysis, the focus of the Activity Theory based analysis included not only the whole-class discussion parts, but also the small-group work parts of the lesson as well, as they provided important information about students' participation in the lesson. Viewing the lesson as an activity, the analysis focused on the nature of participation, identifying the motive of the participants in the activity (i.e., the teacher and the students), and the goals of the actions undertaken by these participants during the activity.

Both the Verbal Analysis and the Activity Theory based analysis indicated that things were not going smoothly in the lesson, and that students did not behave mathematically as desired by the teacher. Yet, the two perspectives provided different interpretations of the situation and of the sources of the problems observed, each interpretation was also supported by current literature. From a cognitive perspective the students' difficulties in the lesson were taken as independent of context. They were interpreted as cognitive difficulties, explained by the cognitive load theory in that the use of multiple external representations (i.e., function representations) is a cognitive obstacle. In contrast, the use of a socio-cultural (Activity Theory) perspective suggested that the teacher and the students participated in the same lesson but in different activities, where different motives, goals, beliefs and norms regarding school mathematics drove and guided them. The teacher's motive was that her students meaningfully learn to solve problems that require the passage from one representation to another. But, the motive of many students was surviving the lesson, expecting the teacher to fulfil her role in helping them have the final answers, not aiming at meaningful learning of the mathematics – a kind of learning that was irrelevant for them.

Are the different interpretations that the two theoretical perspectives provided compatible? Even and Schwarz' (2003) study suggests that they are, and that irrelevance and cognitive obstacles might be interdependent. Yet, determining whether they were indeed interdependent in this particular lesson was unfeasible.

DISCUSSION

The community of researchers in mathematics education acknowledges today the need to use more than one theoretical perspective for better understanding mathematics teaching and learning. Yet, as pointed out by Guba and Lincoln (1994) and clearly exemplified above, the theoretical framework used by the researcher and the findings of the research are interdependent. Therefore, what might be entailed and what might be gained in using more than one theoretical perspective in a single study, as well as the extent to which the interpretations made using particular theoretical approaches are complementary, mutually informing, or incommensurable – need unpacking and careful examination. A promising way to advance understanding about such issues is to conduct empirical studies that purposely analyze one set of data when addressing a single overarching research goal, as

demonstrated above. This is an area for fruitful further work that could profit from stronger and more systematic research programs.

This paper contributes to laying the groundwork for such research programs. It highlights the potential of this kind of studies for advancing the field of mathematics education, and also reveals problems that underlie the challenge to use more than one theoretical perspective in a single study. The main problems emerge from working simultaneously within more than one research paradigm. For example, as illustrated earlier, whereas framing a single wide-ranging research aim for the whole study was successful, no such success occurred when trying to formulate a single research question, as none fitted both theoretical orientations. Similarly, there was also a need to design, for each research question, a research methodology that is aligned with the corresponding theoretical perspective.

The choice of Verbal Analysis as a representative of a cognitive science perspective and Activity Theory as a representative of a socio-cultural perspective necessitates work within two different research paradigms. Each of these research paradigms implies adherence to certain kinds of research questions, particular research methodologies, and confined interpretations of results. Hence, working within different research paradigms in a single study might require the design of parallel lines of inquiry – as illustrated in this paper – each with its own research questions, research methodology, and interpretations of results. Indeed, depending on the theoretical perspectives chosen, it might be possible to formulate the same research question for different lines of inquiry. However, it might not be viable.

Still, even in cases where there is need for more than one line of inquiry – each with distinct research questions, methodology and interpretation of results – working within different research paradigms in a single study requires that the different parallel lines of inquiry share a single over-arching goal and common data sources. Furthermore, an essential part of such studies is the conduction of meta-interpretation of the interpretations of the results obtained from the parallel lines of inquiry, examining their compatibility. Conducting such complex studies is not an easy task. Yet, I believe that it has a great potential to generate novel ways of understanding and thinking about mathematics teaching and learning, not attainable otherwise.

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